# Technical Notes

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## Hypersonic Wind-Tunnel Tests of a Cone Executing Planar and Nonplanar Motion

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THE results of a wind-tunnel test at Mach 14, where a nearly pointed circular cone with a semi-angle of 10 degs performed either planar or nonplanar oscillatory motions with small amplitudes ($\leq$ 3 degs) are summarized here. The specific goal of the experiment was to investigate whether or not aerodynamic coupling moments exist in the case of nonplanar motion. Tobak and Schiff[1] predict that coupling moments in the body axis system, such as a pitching moment due to yawing velocity, and vice versa, must be present in the case of nonlinear aerodynamics. The investigated slender cone model exhibits linear aerodynamics at small angles of attack, and the type of interaction defined by Tobak and Schiff should not be expected. However, a different type of coupling was encountered in the nonplanar motion case that will be discussed here.

### Tests

The cone was attached to the sting of the model support system by means of a two-degree-of-freedom flexure with a cruciform cross section (Fig. 1.). The bosses of one flexure axis were connected to the stationary sting and the angular displacement about this axis is the Euler yaw angle $\psi$. The bosses of the other flexure axis were connected to the cone and the angular displacement about this axis represents the Euler pitch angle $\theta$. In planar motion the model performed free oscillations about either the yaw axis or the pitch axis. In nonplanar motion the model oscillated simultaneously about both flexure axes. The trim angle of attack and sideslip were zero. The oscillation frequencies ($\omega_\theta$ and $\omega_\psi$) were designed to be slightly different. This was accomplished by unequal moments of inertia and spring constants ($I_y \neq I_z$, $k_y \neq k_z$). Upon release from an initial displacement in yaw and in pitch, with zero initial velocities, the model performed a Lissajous-type motion. This motion is schematically shown in Fig. 2. One Lissajous cycle describes the motion going from planar to circular to planar, and then, with reversed precessional direction, to circular and planar. The Lissajous period is $2\pi/(\omega_\psi - \omega_\theta)$. In the wind-tunnel tests the model took approximately 100 oscillations to complete one Lissajous cycle. Thus, the motion changed slowly from planar to nonplanar.

The model was statically balanced to align the center of gravity with the center of the flexure and was also dynamically balanced[2] to align the principal axes of inertia with the body axes. An angular roll misalignment of the principal axes would cause a product of inertia $I_{yz}$ with respect to the body axes. According to Ref. 2, a small $I_{yz}$, which may be two or three orders of magnitude smaller than the transverse moment of inertia, has a strong coupling effect in a two-degree-of-freedom motion.

The flexure was instrumented with strain gages. Approximately 22 angular displacement data were recorded for each cycle of oscillation. A least squares fit to these data was made from the equation

$$\zeta = \zeta_0 e^{\lambda t} \cos(\omega t + \phi) + \zeta_T$$

where $\zeta$ is either the pitch or yaw angle. Thus, amplitude, frequency, and trim angle were obtained as functions of time.

### Results and Discussion

A typical result for a planar free oscillation in pitch is shown in Fig. 3. Excellent repeatability was obtained for several runs. The logarithmic amplitude is seen to be a linear function of time for both wind-on and wind-off conditions. A small change of the oscillation frequency $\omega_\theta$ was observed between the beginning and end of a run. It amounted to less than 1/10 %. The stability derivatives were evaluated by the standard linearized free oscillation data reduction technique. The coefficients $-C_{m_\alpha}=0.23$ and $C_{m_q} + C_{m_{\dot\alpha}} = -2.5$ were determined for a moment center at the 65% axial station. Identical values of the static and dynamic coefficients $C_{n_\beta}$ and $C_{n_r} - C_{n_{\dot\beta}}$ were obtained from planar oscillations about the body $z$-axis.

The pitch amplitudes for a nonplanar motion are shown in Fig. 4. In contrast to Fig. 3, the amplitude envelopes are modulated. Similar modulations were found in the yaw components, however with a phase shift of 180 deg. The frequencies $\omega_\theta$ and $\omega_\psi$ were also modulated. Again, excellent repeatability of the nonplanar motion data was observed for several runs.

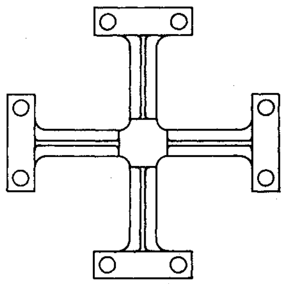

**Fig. 1 Two-degree of freedom flexure.**

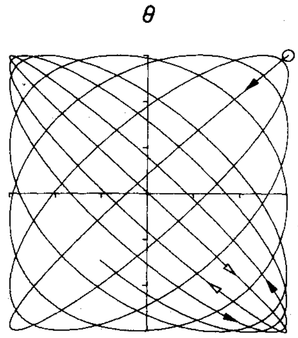


**Fig. 2 Schematic Lissajous-type nonplanar motion.**

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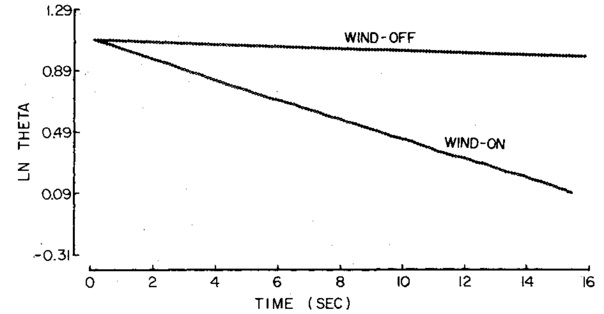


**Fig. 3 Pitch amplitude vs time, planar motion.**

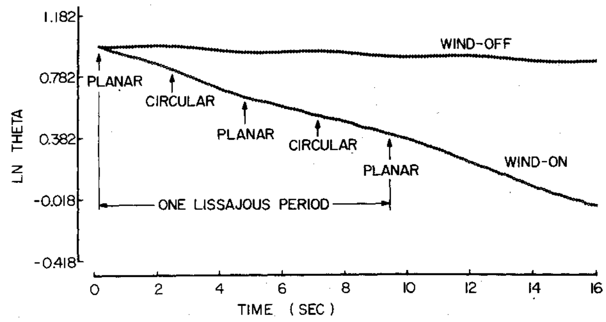


**Fig. 4 Pitch amplitude vs time, nonplanar motion.**

The modulations of the nonplanar motion data were analyzed by means of a computer simulation study. Consider the wind-off data first. The Euler dynamic equation for the moment about the body $y$-axis is given by the expression

$$M = I_y \dot{q} - (I_z - I_x) rp - I_{zx}(r^2 - p^2)$$

$$-I_{xy}(\dot{p} + qr) - I_{yz}(\dot{r} - pq)$$

where for the present test conditions $p = -\dot{\psi}\sin\theta$, $q = \dot{\theta}$, $r = \dot{\psi}\cos\theta$. $M$ is the restoring and damping moment of the flexure. The products of inertia are very nearly zero because the model was dynamically balanced. It was found that the roll coupling term $(I_z - I_x)rp$ can account for the observed modulation of the wind-off data, even though $rp = -\dot{\psi}^2\theta$ is third order while $\dot{q} = \ddot{\theta}$ is first order.

As to the wind-on data in Fig. 4, the exterior moment $M$ in the equation of motion now contains structural and aerodynamic restoring and damping moments. If the roll coupling moment $(I_z - I_x)rp$ is the only coupling term in the pitch equation of motion, then the wind-on data should modulate sinusoidally, as is the case for the wind-off data. However, this is not true, as Fig. 4 shows a "flat spot" at the time interval from about 6-10 sec. This indicates that another coupling moment is present.

Although the wind-on modulation is small (approximately 0.1°) the slope of the logarithmic envelope varies by a factor of two. This then raises the question of how to evaluate the aerodynamic damping. It has been suggested [3,4] that the aerodynamic damping may vary with a change of the motion pattern. If this were true, the envelope should modulate with a frequency twice that of the Lissajous frequency as the model goes through two circular motion patterns for each Lissajous cycle. Figure 4 shows that this is not the case. In fact, the period of the wind-on modulation equals the Lissajous period. It was concluded that the aerodynamic damping was not affected by the motion pattern and the derivaties were determined by averaging the data over a full-beat period. The results obtained in this manner were identical with those for planar motion. This result is in agreement with that predicted by Tobak and Schiff,[1] who showed that the stability derivatives for planar and nonplanar motions will be different for a body of revolution only when the motion is governed by nonlinear aerodynamics.

In the search for an explanation of the wind-on modulations (seen in Fig. 4), the possibility of experimental anomalies was considered first. It was thought that a slight warping of the model due to asymmetric aerodynamic heating, sting vibration, support interference, or aerodynamic loading on the flexure, either alone or in combination, might produce the wind-on modulations. It was experimentally established that none of these conditions accounted for the observed modulations.

Next, a computer study was performed to investigate the effect of nonlinear coupling terms. Data were generated with various nonlinear terms, including those suggested by Tobak and Schiff.[1] It was found that the nonlinear coupling terms did not produce the low-frequency modulation observed in the tests. Finally, linear coupling terms were investigated. The equations were of the form

$$I_y\ddot{\theta} + D_\theta\dot{\theta} + k_\theta\theta = A\ddot{\psi} + B\dot{\psi} + C\psi$$

$$I_z\ddot{\psi} + D_\psi\dot{\psi} + k_\psi\psi = A\ddot{\theta} + B\dot{\theta} + C\theta$$

where $D$ and $k$ are the damping and restoring coefficients. It was found that $B$ was ineffective in duplicating the modulations, whereas values for $A$ and /or $C$ were found which produced the low-frequency modulation observed in the wind-on data.

These results infer the existence of linear aerodynamic coupling for nonplanar motions. For planar motions, the existence of any aerodynamic coupling seems physically impossible and indeed the planar wind-on data of Fig. 3 do not show any coupling effects. While the inference of aerodynamic coupling parallels Jaffe's's[4] speculation regarding inadequacies in the aerodynamic force model, further studies must be carried out before the hypothesis of aerodynamic coupling for nonplanar motion can be accepted.

## Conclusions

Experimental tests were carried out for a 10° nearly pointed cone performing either planar or nonplanar motions with small amplitudes.[5] The stability derivatives ($C_{m_\alpha}$ and $C_{m_q} + C_{m_{\dot{\alpha}}}$) were evaluated by the standard free oscillation data reduction technique and found to be identical for planar and nonplanar motion. This result is in agreement with the analysis of Tobak and Schiff,[1] but is not in accord with the experimental results of Jaffe.[4] Jaffe determined that the damping derivative depended upon the type of motion.

A modulation was observed in the wind-on data. Experimental tests showed that this modulation was not caused by experimental anomalies. Computer simulation studies using nonlinear coupling terms could not reproduce this modulation, whereas either a linear acceleration coupling term or a linear restoring coupling term would account for the modulation.

## References

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